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Interval analysis for the sensors faults detection of a nonlinear system

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***Abstract*— The work presented in this paper deals with the sensors faults detection using the interval analysis. The presented method is then applied to the nonlinear systems described by multiple model structures; the interval analysis is used to estimate the output in the case of system measurement uncertainties. The proposed technique is insensitive to measurement uncertainties and highly reliable in case of a fault affecting the system.**

Keywords: diagnostic, Interval analysis, sensors faults, uncertain system

I. INTRODUCTION

An accurate representation of the system model is the challenge faced by the scientific community. Several methods have been proposed to model the behavior of systems in order to ensure their supervision. Indeed all faults must be detected to ensure the materials and human security. Hardware redundancy is often considered to detect and locate faults solution, or any material is ideal. Each instrument has an uncertainty which is often indicated by the manufacturer.

In this work a method for diagnosis of nonlinear systems with measurement uncertainties is proposed. The presented method is based on the interval analysis as tools for state estimation, detection and fault location.

Generally, faults can be detected by performing consistency checks verifying the adequacy of the information provided by the model and the sensors. A fault is detected when the residue goes away of zero (for filter-based methods [1], observers [2] or space parity [3]) or when the parameters derived abnormally in the methods of parametric estimations [4].

The interval analysis permits to define an envelope in which the fault is considered as non-existent and any deviation of measurement inside this envelope is considered as normal. This approach has been explored by Adrot [5, 6] and applied to a mechatronic system by Letellier [7]

II. INTERVAL ANALYSIS

The state estimation of a system is a problem solved if the system has constant parameters or if it is subject to known statistical disturbances characteristics [8]. In general, estimation techniques are based on the knowledge of mathematical model of the system. In addition, the information collected about a system are are given by sensors whose reliability can be questioned [9], which can causes disruptive phenomena that undermine the ability of models to accurately reflect the systems behavior. the used method for changes in system parameters considers bounded perturbations whose boundaries are known in advance.

In general cases disturbances are unknown, so it is impossible to determine the actual system state. The solution will be in the estimation of a domain which these states belong in. This problem is known as "set-valued state estimation" [10-13]and it can be formulated by considering the following model systems [8]:

$$S\begin{cases} x(k+1) = f\left(x(k), u(k), v(k)\right) \\ y(k) = h\left(x(k)\right) + w(k) \end{cases} \qquad (1)$$

Where $x \in \mathbb{R}^n$ is a system state, $u \in \mathbb{R}^q$ is the input, $y \in \mathbb{R}^m$ is the observed output; $v(k)$ et $w(k)$ are bounded disturbances or uncertainties. The main goal is to estimate the bounds $\left(x^-(k) \quad x^+(k)\right)$ of state $x(k)$,knowing the perturbations limits $\left(v^-(k) \quad v^+(k)\right)$ and $\left(w^-(k+1) \quad w^+(k+1)\right)$, the model S, and the output and input measures , we wish to At time k, the allowable state domain is defined using the bounds $\left(x^-(k) \quad x^+(k)\right)$, of the, for example in the form of box:

$$D_{x,k} = \left\{ x / x^-(k) \le x(k) \le x^+(k) \right\} \qquad (2)$$

At the time (k+1), the new domain $D_{x,k+1}$ is constructed from previous domains $D_{x,k}, \cdots, D_{x,0}$ of the new measure $u(k), y(k+1)$ and the bounds $\left(v^-(k) \quad v^+(k)\right)$ and $\left(w^-(k+1) \quad w^+(k+1)\right)$. For nonlinear systems, the analytical determination of the domain $D_{x,k+1}$ is difficult and we must resort to numerical methods for evaluation. The case of linear systems is easier; the states domain can be represented by a polytope.

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III. PROBLEM FORMULATION

The hypothesis of measurement precision is not always valid because the sensors cannot be in most cases perfectly accurate. This technological imprecision is often mentioned by the manufacturer [14].

Consider the nonlinear system with uncertain outputs described by a multiple model with decoupled structure based on measurable decision variables $\xi(k)=u(k)$ [15-17]

$$x_i(k+1)=A_i x_i(k)+B_i u(k) \quad (3)$$

$$y_i(k)=C_i(\eta_{C_i}(k))x_i(k) \quad (4)$$

$$y_m(k)=\sum_{i=1}^{M}\mu_i(u(k))y_i(k) \quad (5)$$

With $x_i\in\mathbb{R}^n, u\in\mathbb{R}^q$ *et* $y_i\in\mathbb{R}^m$ are respectively the state vectors, the input and output of the i^{th} sub-model. $\mu_i(u(k))\in[0,1]$ represent the activation functions depending on the input.

$C_i(\eta_{C_i}(k))\in\mathbb{R}^{m\times n}$ is the uncertain matrix of the i^{th} sub-model defined by:

$$\begin{aligned}&C_i(\eta_{C_i}(k))=C_i^0\pm\Delta_{C_i}\otimes\eta_{C_i}\\&C_i^0\in\mathbb{R}^{m\times n},\Delta_{C_i}\in\mathbb{R}^{m\times n},\eta_{C_i}\in\mathbb{R}^{m\times n}\end{aligned} \quad (6)$$

Where $\otimes$ is the operator of multiplication of two matrices element by element. C_i^0 is the nominal matrix of $C_i(\eta_{C_i}(k))$. Δ_{C_i} is the uncertainties amplitude matrix the of the elements of $C_i(\eta_{C_i}(k))$. η_{C_i} is the matrix consisting of bounded and standard variables of i^{th} sub-model. $i\in 1,\ldots,M$

The uncertain matrix $C_i(\eta_{C_i}(k))$ can be represented by their extreme limits, where it has the following form:

$$C_i(\eta_{C_i}(k))\in[C_i]=\left[\underline{C}_i,\overline{C}_i\right],\quad \left|\eta_{C_i}(k)\right|\le 1$$

Recall that the relations (3), (4) and (5) describe a nonlinear system represented by M uncertain sub-models related by weighting functions $\mu_i(u(k))$ satisfying the properties of convexity $\sum_{i=1}^{M}\mu_i(u(t))=1$, and $0\le\mu_i\le 1$ [18, 19]

IV. STATE ESTIMATION

From equation (4), an estimated can be provided:

$$\hat{x}_i(k)=C_i^{-1}(\eta_{c_i}(k))y_i(k) \quad (7)$$

Consider that the uncertain matrices $C_i(\eta_{C_i}(k))$ are square, observable, and bounded $C_i(\eta_{C_i}(k))\in[C_i]=\left[\underline{C}_i,\overline{C}_i\right]$, they can be replacing in (7) by their lower and upper bounds:

$$[\hat{x}_i^{(1)}(k)]=[C_i]^{-1}y_i(k) \quad (8)$$

In (8) the estimated states $[\hat{x}_i^{(1)}(k)]$ are obtained from a measurement system, as originally assumed valid.

To solve this equation, there are many methods such as Gaussian elimination, the Gauss-Seidel iteration and the Krawczyk iteration [20, 21] which have already been used by Alhaj-Dibo[14, 22] to solve the same problem in the case of linear systems

The optimal solution of $[\hat{x}_i^{(1)}(k)]$ is the intersection of the three options provided by the three methods mentioned above.

$$[\hat{x}_i^{(1)}(k)]=[\hat{x}_{i_G}^{(1)}(k)]\cap[\hat{x}_{i_{GS}}^{(1)}(k)]\cap[\hat{x}_{i_K}^{(1)}(k)] \quad (9)$$

In the system equation (3) $[x_i(k)]$ can be replaced by their estimated intervals calculated in (8), the estimated variables can be computed at time *k* and *k +1*

$$[\hat{x}_i^{(2)}(k+1)]=A_i[\hat{x}_i^{(1)}(k)]+B_i u(k) \quad (10)$$

In this case, the estimates $[\hat{x}_i^{(2)}(k+1)]$ are obtained from the system model (8) and measures (10).

It can be concluded that the estimates of the state variables at time *k +1* are calculated twice (from the system at time *k* and from measurements at time *k +1*). So the final local estimates $[\hat{x}_i(k+1)]$ are obtained by performing the intersection between the two estimates

$$\begin{aligned}&[\hat{x}_i(k+1)]=[\hat{x}_i^{(2)}(k+1)]\cap[\hat{x}_i^{(1)}(k+1)]\\&i=\{1\ldots M\}\end{aligned} \quad (11)$$

The state estimation, in the case of multiple models with uncertain measures, is locally on each sub-model.

V. FAULT DETECTION AND LOCATION

At this stage, the whole system is considered faulty if one fault can be detected on one of the sub models.

For fault location, residuals must be generated for the fault detection and location. So the local coherence between the estimates $[\hat{x}_i^{(2)}(k+1)]$ and $[\hat{x}_i^{(1)}(k+1)]$ at every time for $i\in\{1...M\}$ will be tested

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Consider that at the instant k measurements are safe. In this case the estimate obtained from the system model and measurement model at the same time $[\hat{x}_i^{(2)}(k+1)]$ is coherent with the system model, which allows considering it as a reference in the comparison with the estimated obtained from measurements at time $k+1$ $[\hat{x}_i^{(1)}(k+1)]$.

From the foregoing, one can judge the state of the local estimated, in the case of the safe functioning and faulty functioning

$[\hat{x}_i(k+1)] = [\hat{x}_i^{(2)}(k+1)] \cap [\hat{x}_i^{(1)}(k+1)]$ For the safe functioning (12)

$[\hat{x}_i(k+1)] = [\hat{x}_i^{(2)}(k+1)]$ For the faulty functioning (13)

Referring to, the estimated interval calculated from equation (11) is analyzed. Two cases are possible:

- If $[\hat{x}_i(k+1)] = [\hat{x}_i^{(2)}(k+1)] \cap [\hat{x}_i^{(1)}(k+1)] \neq \varnothing$; the estimated intervals $[\hat{x}_i^{(2)}(k+1)]$ and $[\hat{x}_i^{(1)}(k+1)]$ are coherent, which sub-model measures, at the same time $k+1$, are safe.
- If $[\hat{x}_i(k+1)] = [\hat{x}_i^{(2)}(k+1)] \cap [\hat{x}_i^{(1)}(k+1)] = \varnothing$; the two estimated intervals $[\hat{x}_i^{(2)}(k+1)]$ and $[\hat{x}_i^{(1)}(k+1)]$ are not coherent where the measures of the hole system at the same time k +1, contain at minus one fault. The ith sub-model is then used to locate the fault.

In this case, the safe state variables values are considered belong to the reference $[\hat{x}_i^{(2)}(k+1)]$ only, and it is not possible to consider the estimated interval, obtained from the system model and measurements at time $k+2$ $[\hat{x}_i^{(2)}(k+2)]$, as a reference to compare with $[\hat{x}_i^{(1)}(k+2)]$. Fault must be corrected first.

Using the system model (3), the states variables are replaced by their estimated considered as reference to the same time $[\hat{x}_i^{(2)}(k+1)]$ hence :

$$[\hat{x}_i^{(2)}(k+2)] = A_i[\hat{x}_i^{(2)}(k+1)] + B_i u(k+1) \qquad (14)$$

To identify measures affected by a fault. Residues intervals are generated for each sub-model $[r_i(k+1)]$. From equation (4), replaces the state variable at time$(k+1)$by their estimated considered as reference at the same instant $[\hat{x}_i^{(2)}(k+1)]$ and the matrix $C_i(\eta_C(k))$ by its value interval $[C_i]$ whence:

$$[r_i(k+1)] = [C_i][\hat{x}_i^{(2)}(k+1)] - y_i(k+1) \qquad (15)$$

A measure is affected by a fault if the corresponding residue is abnormal i.e. the residue interval corresponding does not contain the value 0.

VI. Example

Consider a nonlinear system with three uncertain outputs described by the multiple model (3)(4)(5) with M=2 submodels were

$$A_1 = \begin{bmatrix} 0.51 & -0.18 & -0.23 \\ 0.12 & 0.81 & -0.1 \\ -0.29 & -0.31 & 0.55 \end{bmatrix} \quad A_2 = \begin{bmatrix} -0.44 & 0.29 & 0.2 \\ 0.325 & 0.32 & -0.2 \\ 0.09 & -0.099 & 0.35 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.5 \\ -0.28 \\ 1.1 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0.78 \\ -1.25 \\ 1.4 \end{bmatrix}$$

$$C_1(\eta_{C_1}(k)) \in [\underline{C}_1 \quad \overline{C}_1] = \begin{bmatrix} [-5.05 \quad -5.25] & [-1.78 \quad -1.065] & 1.12 \\ [0.89 \quad 1.12] & [0.16 \quad 0.22] & [-5 \quad -4.5] \\ [-2 \quad -1.75] & [-1.83 \quad -1.3] & [0.8 \quad 1] \end{bmatrix}$$

$$C_2(\eta_{C_2}(k)) \in [\underline{C}_2 \quad \overline{C}_2] = \begin{bmatrix} [-6.75 \quad -6.2] & [-2.5 \quad -1.28] & [1 \quad 1.08] \\ [1.85 \quad 2.05] & [0.14 \quad 0.15] & [-5.2 \quad -4.5] \\ [-2.1 \quad -2] & [-1.31 \quad -1.3] & [1.91 \quad 2.09] \end{bmatrix}$$

Activation functions $\mu_i(u(k))$ should be normalized to ensure the convexity property. They are represented in Figure 1

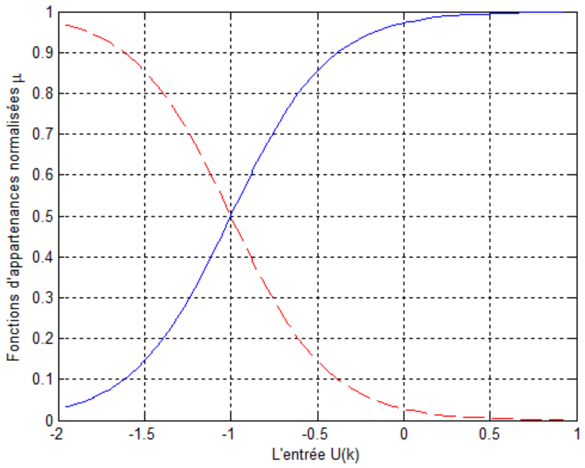

Fig.1: Activations functions normalized depending on the input.

A control U (k) with uniform distribution is applied to the system

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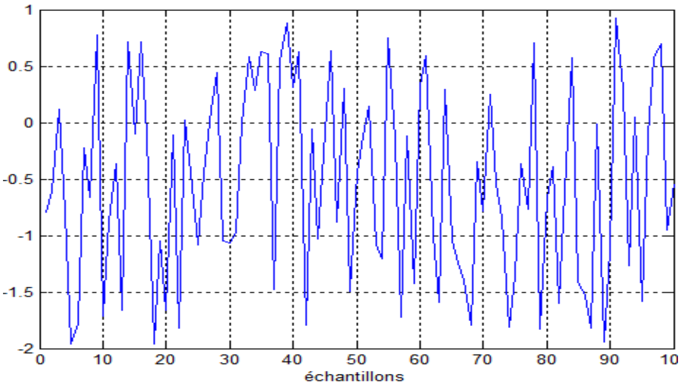

Fig.2: control U (k) applied to the system.

Assume that a fault affects the three sensors at different times, represents the safe system outputs and the faulty outputs at the instants [10, 20], [40, 50] and [60, 70].

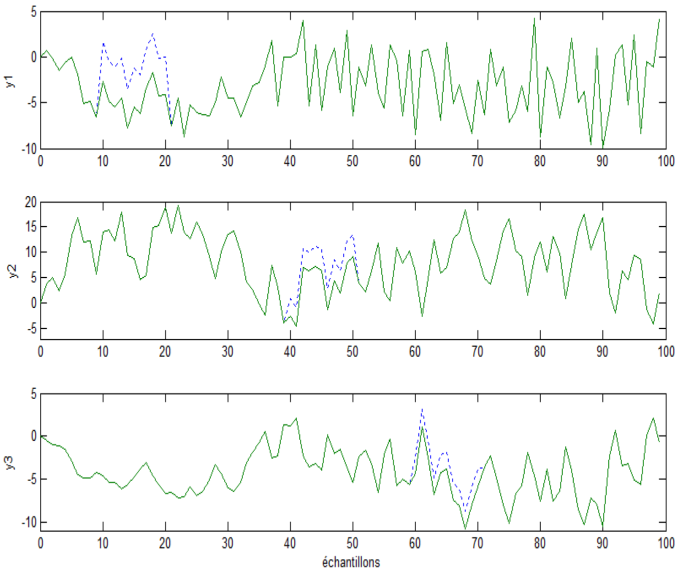

Fig.3: outputs obtained with their corrections.

Referring to relations (8), (10) and (11), generating an estimated interval of the state variables. This is considered as limited by the upper bound and the lower bound of the estimated area. From the results one can determine an output estimate. The system measures should be included in the output domain estimation. Means that if the measurements exceed the upper or lower bound of the estimation domain, the sensor will be declared faulty. Figure 4 shows estimates of the three outputs as a closed area, and the three outputs measured (dashed line).

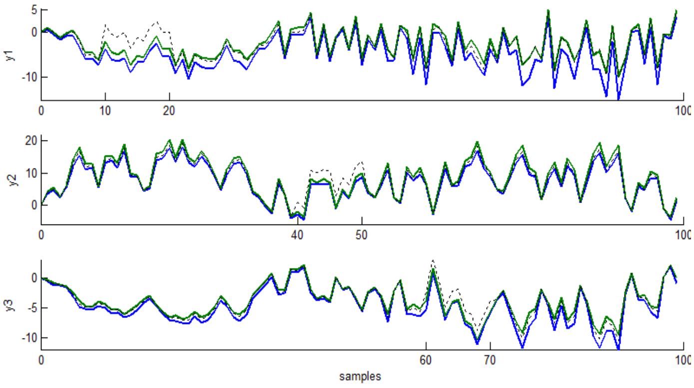

Fig.4: Estimated and measured outputs.

Note that, at the instants [10, 20], [40, 50] and [60, 70], the measurements y1, y2 and y3 exceed the limit of the estimation domain, where a fault is detected in the precise moment of the output concerned. To better locate faults affecting the system, the indicator signals that are also called residuals are generated. These signals will be in the form of a sealed envelope must contain the value zero as the system is not affected by any fault. Otherwise, zero will be located outside the envelope. In figure 5, the residuals envelopes of the three outputs are represented.

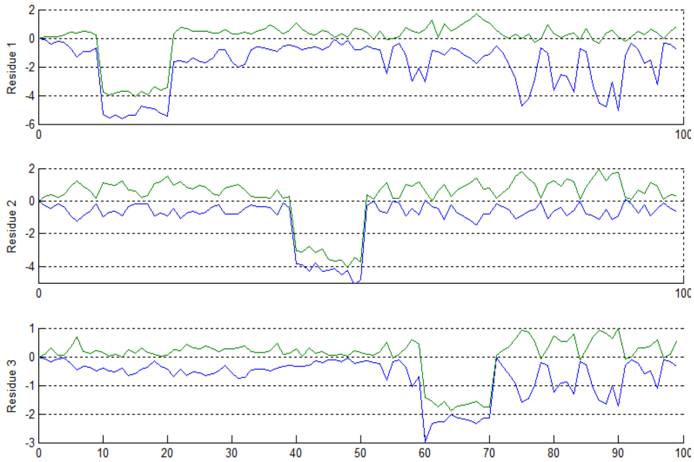

Fig.5: residues.

Note that each fault t impacts its residual at the right time. In this way, we get to perfectly locate defects. Iterating between 10 and 20, it has a fault that affects the first sensor, between 40 and 50; another fault affects the second output, and between 60 and 70 a third fault taints the last output.

VII. Conclusion

No sensor is considered perfect [23]. To ensure a robust diagnosis, this work, characterize the measurement uncertainties using the interval analysis. Our contribution in this work is to use this method for the case of nonlinear systems described by multiple models structures with uncertain outputs. The used methods allow also estimating the system output and to detect

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and locate faults in case of their presence. This method is robust to the measurement uncertainties.

The presented method is an effective technique with a low cost of calculation and it can be generalized to the case of modeling or command uncertainties

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